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Elastic wave omnidirectional absorbers designed by transformation method

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A conformal mapping is proposed to design with transformation method broadband elastic wave omnidirectional absorbers. This conformal mapping transforms the infinity of the virtual space into the origin of the physical space and keeps the transformed material isotropic. The material realization of such elastic wave absorber is proposed and validated by numerical simulation. Different from the existing procedure based on Hamiltonian optics, the proposed method is a more general one and can be applied to any physical process where the transformation method is valid. It also includes the existing techniques as the special cases for electromagnetic and acoustic waves. © *2012 American Institute of Physics*. [http://dx.doi.org/10.1063/1.4740077]

Energy absorption and collection are important issues in various engineering applications; energy flows in these cases are usually directed into some regions where they are absorbed or taken away. An omnidirectional absorber, also called "optical black-hole," is recently designed based on Hamiltonian optics[1] and subsequently validated experimentally through different procedures.[2–4] By analogy, similar acoustic devices are also designed both analytically[5] and experimentally.[6] However, this analogy is not available for elastic waves in solids, which represent vast areas of applications for solid structures instead of liquids for acoustic waves; therefore, it is interesting to develop a method to design elastic wave absorbers. The recent transformation method can systematically establish the relation between material distribution and function of device, and it is applied for different types of waves, such as electromagnetic,[7–9] acoustic,[10–12] matter waves,[13] and even surface water waves.[14] It is also extended approximately to elastic waves[15] and fluids dynamics.[16] Therefore, it is natural to imagine if we find an appropriate mapping representing the function of a "black-hole" device, elastic wave omnidirectional absorbers can then be designed directly with transformation method. This is main objective of the work.

In this letter, a general design approach for elastic waves "black-hole" absorption device is proposed with the help of transformation method. A conformal mapping is found to construct the geometry space of such device; the necessary material distribution of the device is obtained with the corresponding transformation relation. The designed elastic wave absorber and its corresponding prototype are then validated by numerical simulations. Finally, we also discuss the versatility of the proposed method, which can include the existing ones as special cases.

We start by considering the following conformal mapping:

$$w = Az^n \; (A \neq 0,\; n < 0), \tag{1}$$

in which $z = x + iy$ represents the virtual space, $w = x' + iy'$ the physical space, and A is an arbitrary constant. When $n \in (-\infty, 0)$, a flat virtual space is mapped into a curved one, and at the same time, the infinity of the virtual space is mapped into the origin of the physical space (see Fig. 1 as an example). Although not intuitive, this fact reveals its potential in designing "black-hole" devices. Moreover, the larger the n is, the bigger the "gravitational force" will be.

The conformal mapping keeps the isotropic property of the transformed material if the material in the virtual space is also isotropic.[8,17,18] In this context, the Jacobian determinant J is used as the only variable connecting the space and material. Specifically for the mapping (1), the Jacobian determinant is obtained as

$$J = An\left(\frac{1}{A}r'\right)^{\frac{2(n-1)}{n}}, \tag{2}$$

where $r' = \sqrt{x'^2 + y'^2}$ is the polar coordinate of the physical space.

The outer boundary of the device is required to match with the surrounding space, to this end, from the mapping (1); there is no "transformation" (or more precisely, there is no principal stretch[15]) between the virtual space and physical space at the position where $J = 1$. Therefore, we use $J = 1$ as a constraint condition to determine the outer boundary of the device. After simple calculation, a circular geometry of the device with a radius R' is defined as

$$R' = |A(An)^{\frac{n}{2(1-n)}}|. \tag{3}$$

Based on this result, inserting Eq. (3) into Eq. (2), a more concise variation of the Jacobian determinant as function of R' and n is found as

$$J = \left(\frac{R'}{r'}\right)^{-2+\frac{2}{n}},\; n \in (-\infty, 0). \tag{4}$$

With Eq. (4), an elastic wave "black-hole" device can be designed with help of transformation elastodynamics. As been reported in Ref. 18, there are different forms of the transformation relations in transformation elastodynamics for a conformal mapping. One simple choice is to make the mass density unchanged

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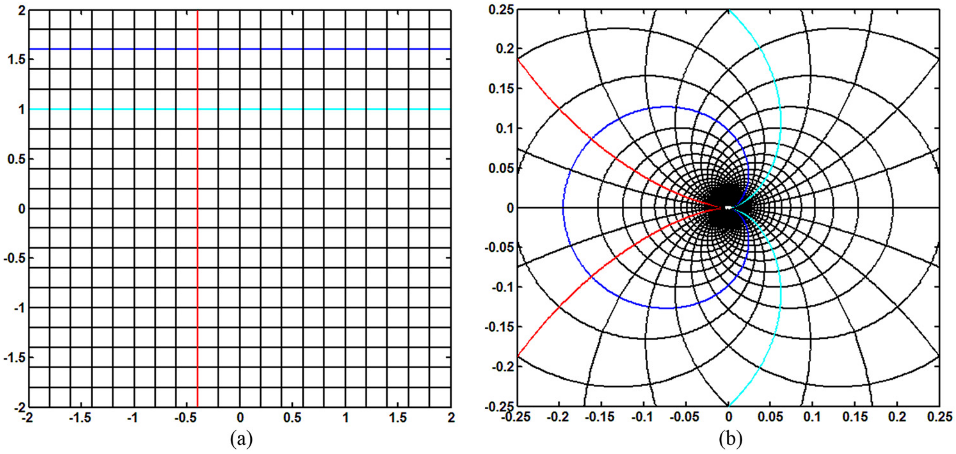

FIG. 1. Schematic diagram of a flat virtual space $z = x + iy$ (a) and a curved physical space $w = x' + iy'$ (b) transformed by the mapping $w = 1/2z^2$.

$$\rho' = \rho,\ E' = JE,\ \upsilon' = \upsilon. \tag{5}$$

where ρ is mass density, E is Young's modulus, and υ is Poisson's ratio. The notations with and without prime represent the material parameters in the virtual and the physical space, respectively. With the above results, an elastic wave "black hole" is designed by inserting Eq. (4) into Eq. (5)

$$\rho' = \rho,\ E' = \left(\frac{R'}{r'}\right)^{-2+\frac{2}{n}} E,\ \upsilon' = \upsilon,\ (r' \le R'). \tag{6}$$

To avoid singularity and large gradient of material parameters, we replace the central part of the device ($r' \le R_c{}'$) with a homogeneous material[1–6]

$$\rho' = \rho,\ E' = \left(\frac{R'}{R_c{}'}\right)^{-2+\frac{2}{n}} E(1+i\delta),\ \upsilon' = \upsilon,\ (r' \le R_c{}'). \tag{7}$$

FEM simulations for this elastic wave "black-hole" device are carried out. The parameters of the devices are set to be $R' = 12\,\text{mm}$, $R_c{}' = 4\,\text{mm}$, $\delta = 0.6$, and $n = -\infty$ in order to facilitate the comparison with the previous works.[1,5] The background space is set to be aluminum foam ($\rho = 0.4\,\text{kg/m}^3$, $E = 2\,\text{GPa}$, $\upsilon = 0.27$, and $\delta = 0.003$ (Ref. 19)). It is shown in Fig. 2(a) that a Gaussian longitudinal elastic wave beam (P-wave) with a frequency $f = 800\,\text{kHz}$ impinging on the device is bended into the shell region and then absorbed by the core. The similar phenomenon is also observed for the shear wave (S-wave) case, as shown in Fig. 2(b). Compared with previous works,[1,5] here we obtained the identical patterns, although the waves are totally different.

To show the flexibility brought by transformation method, in the second example we choose arbitrarily different relations[18]

$$\rho' = J^{-\frac{1}{2}}\rho,\ E' = J^{\frac{1}{2}}E,\ \upsilon' = \upsilon. \tag{8}$$

Inserting Eq. (4) into Eq. (8), the material distributions of the elastic wave "black-hole" are derived as

$$\rho' = \left(\frac{R'}{r'}\right)^{1-\frac{1}{n}} \rho,\ E' = \left(\frac{R'}{r'}\right)^{-1+\frac{1}{n}} E,\ \upsilon' = \upsilon,\ (r' \le R'). \tag{9}$$

Also, the core region ($r' \le R_c{}'$) is set to be

$$\rho' = \left(\frac{R'}{R_c{}'}\right)^{1-\frac{1}{n}} \rho,\ E' = \left(\frac{R'}{R_c{}'}\right)^{-1+\frac{1}{n}} E(1+i\delta),\ \upsilon' = \upsilon,\ (r' \le R_c{}'). \tag{10}$$

As required, the material parameters of the elastic "black-hole" device are all isotropic, so the implementation turns to be very simple. In the following, a detailed scheme to construct the above elastic wave "black-hole" will be examined. In the case where the background material is still an aluminum foam and the parameters of the "black-hole" device are $R' = 1\text{m}$, $R_c{}' = 0.5\,\text{m}$, $\delta = 0.6$, and $n = -1$, the

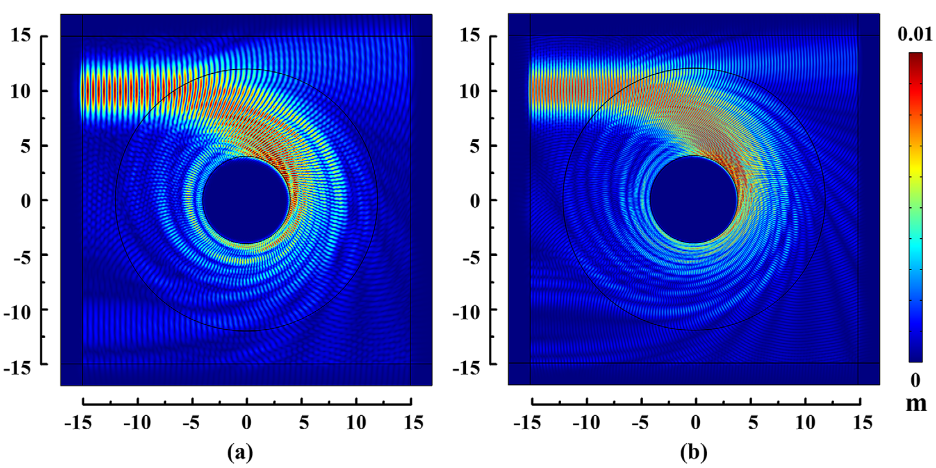

FIG. 2. (a) The total displacement field when a Gaussian longitudinal elastic wave (P-wave) beam ($f = 800\,\text{kHz}$) is incident on the elastic wave "black-hole." (b) The same as (a) but in a shear wave (S-wave) case.

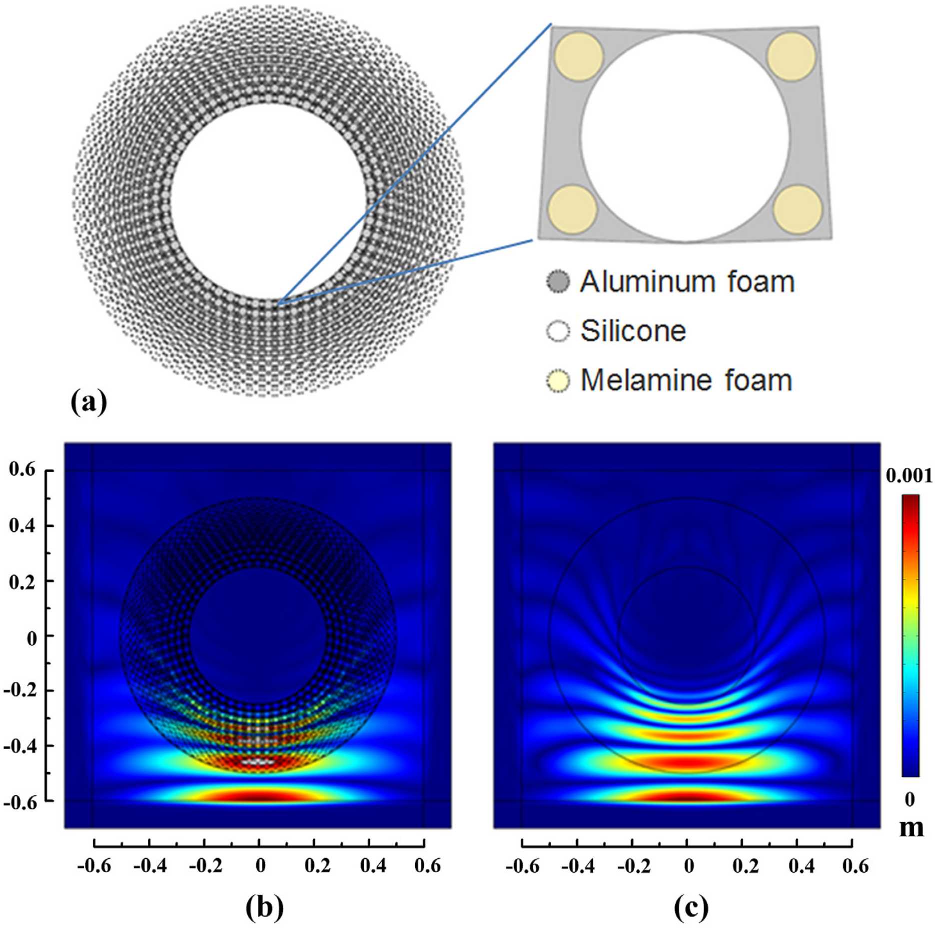

FIG. 3. (a) The implementation scheme of an "elastic wave black-hole." The "black-hole" region is spliced by the cell elements consisting of aluminum foam, silicone and melamine foam. The total displacement field of the designed "black-hole" device (b) shows the incident S-wave beam was bended and absorbed in the device, which has a similar result with that of the continuous model (c).

"black-hole" region can be discretized into10 layers with the same thickness. Each layer is spliced by the cell element as shown in Fig. 3(a). In the quasi-static approximation, the effective material parameters follow the mixing rule; therefore, the required material parameters in each layer are fulfilled by adjusting the radius of the circular inclusion in each cell element. As the matrix is an aluminum foam, we choose silicone ($\rho = 2.3\,\mathrm{kg/m^3}$, $E = 0.05\,\mathrm{GPa}$, $\upsilon = 0.47$, and $\delta = 1.6$ (Ref. 19)) and melamine foam ($\rho = 0.012\,\mathrm{kg/m^3}$, $E = 0.0004\,\mathrm{GPa}$, $\upsilon = 0.2$, and $\delta = 0.5$ (Ref. 19)) as the materials for the inclusions, in order to adjust two effective material parameters (ρ' and E') simultaneously. The Poisson's ratio υ is not controlled in the following. FEM simulations are carried out on this discrete model to illustrate the effectiveness of the prototype of the device. To reduce the computation cost, a homogeneous core is set to be an ideal material followed from Eq. (10); it can also be experimentally realized by the similar cell element as shown in Fig. 3(a). As illustrated in Fig. 3(b), a S-wave beam with $f = 5\,\mathrm{kHz}$ is bended and absorbed by the proposed device, and the patterns are almost the same as the ideal continuous model with $\delta = 0.15 \times (\frac{R'}{r'})^2$ in the shell region, which is shown in Fig. 3(c). Here we have to note that in this scheme, the bandwidth is only limited by the cell structure since the quasi-static condition is used to derive the effective material parameters; this requires the wavelength to be large enough compared to the scale of the cell element. To enlarge the bandwidth, a model with relatively more layers and relatively smaller cell elements is required, which we think is not a major problem for the manufacturing technique of today. In addition, as we have already discussed above, for such broadband elastic wave omnidirectional absorber, we have a sufficient flexibility in design process. For example, with the material distribution (7), a similar device constituted by only two kinds of materials (one as background material and the other as inclusion) can also be implemented.

Although the main purpose of this letter is to design ***elastic wave*** absorbers, the proposed method can also be applied to design "black-hole" devices for other waves. For example, in "transformation optics", the transformation relations for 2-D TM problems are given by

$$\boldsymbol{\mu}' = \frac{\mathbf{A}\mathbf{A}^{\mathrm{T}}}{J}\mu,\ \varepsilon' = \frac{1}{J}\varepsilon, \tag{11}$$

where $\mathbf{A}$ is the Jacobin matrix of the mapping. A non-magnetic "optical black-hole" is then designed by inserting the Cauchy-Riemann condition and Eq. (4) into Eq. (5)

$$\mu' = \mu,\ \varepsilon' = \left(\frac{R'}{r'}\right)^{2-\frac{2}{n}}\varepsilon. \tag{12}$$

Similarly, in "transformation acoustics", with the transformation relation reported in Ref. 10

$$\boldsymbol{\rho}^{-1\prime} = \frac{\mathbf{A}\mathbf{A}^{\mathrm{T}}}{J}\rho^{-1},\ \kappa' = J\kappa, \tag{13}$$

the "acoustic black hole" with the following material parameters

$$\rho' = \rho,\ \kappa' = \left(\frac{R'}{r'}\right)^{-2+\frac{2}{n}}\kappa, \tag{14}$$

is obtained; they are the same as those reported before.[1,5] Besides these reported results, devices with the same function but different type of material distribution can also be designed due to no unique transformation relations in transformation acoustics.[20,21] More importantly, with increasing applications of transformation method, the proposed procedure can be used to design omnidirectional absorbers for any other physical processes where transformation method can be applied.

In conclusion, based on a conformal mapping and transformation method, we have proposed a general approach to design elastic wave "black-hole" absorbers. A prototype of such elastic wave absorber is also proposed and verified by numerical simulations. The proposed method is general one, and the existing methods are covered as special cases. It can also be applied to any physical processes where the transformation method holds. The possible applications in elastic wave absorption and energy harvesting can be anticipated.

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[1] E. E. Narimanov and A. V. Kildishev, "Optical black hole: Broadband omnidirectional light absorber," Appl. Phys. Lett. **95**, 041106 (2009).
[2] Q. Cheng, T. J. Cui, W. X. Jiang, and B. G. Cai, "An omnidirectional electromagnetic absorber made of metamaterials," New J. Phys. **12**, 063006 (2010).
[3] W. Lu, J. Jin, Z. Lin, and H. Chen, "A simple design of an artificial electromagnetic black hole," J. Appl. Phys. **108**, 064517 (2010).
[4] J. Zhou, X. Cai, Z. Chang, and G. K. Hu, "Experimental study on a broadband omnidirectional electromagnetic absorber," J. Opt. **13**, 085103 (2011).
[5] R. Li, X. Zhu, B. Liang, Y. Li, X. Zou, and J. Cheng, "A broadband acoustic omnidirectional absorber comprising positive-index materials," Appl. Phys. Lett. **99**, 193507 (2011).
[6] A. Climent, D. Torrent, and J. Sanchez-Deseha, "Omnidirectional broadband acoustic absorber based on metamaterials," Appl. Phys. Lett. **100**, 144103 (2012).
[7] J. B. Pendry, D. Schurig, and D. R. Smith, "Controlling electromagnetic fields," Science **312**, 1780 (2006).
[8] U. Leonhardt, "Optical conformal mapping," Science **312**, 1777 (2006).
[9] A. Greenleaf, M. Lassas, and G. Uhlmann, "On nonuniqueness for Calderón's inverse problem," Math. Res. Lett. **10**, 685 (2003).
[10] H. Chen and C. T. Chan, "Acoustic cloaking in three dimensions using acoustic metamaterials," Appl. Phys. Lett. **91**, 183518 (2007).
[11] S. A. Cummer and D. Schurig, "One path to acoustic cloaking," New J. Phys. **9**, 45 (2007).
[12] A. N. Norris, "Acoustic cloaking theory," Proc. R. Soc. A **464**, 2411 (2008).
[13] S. Zhang, D. A. Genov, C. Sun, and X. Zhang, "Cloaking of matter waves," Phys. Rev. Lett. **100**, 123002 (2008).
[14] M. Farhat, S. Enoch, S. Guenneau, and A. B. Movchan, "Broadband cylindrical acoustic cloak for linear surface waves in a fluid," Phys. Rev. Lett. **101**, 134501 (2008).
[15] J. Hu, Z. Chang, and G. K. Hu, "Approximate method for controlling solid elastic waves by transformation media," Phys. Rev. B **84**, 201101(R) (2011).
[16] Y. A. Urzhumov and D. R. Smith, "Fluid flow control with transformation media," Phys. Rev. Lett. **107**, 074501 (2011).
[17] J. Li and J. B. Pendry, "Hiding under the carpet: A new strategy for cloaking," Phys. Rev. Lett. **101**, 203901 (2008).
[18] Z. Chang, J. Hu, G. K. Hu, R. Tao, and Y. Wang, "Controlling elastic waves with isotropic materials," Appl. Phys. Lett. **98**, 121904 (2011).
[19] Detailed material parameters can be found in Cambridge Engineering Selector.
[20] J. Hu, X. N. Liu, and G. K. Hu, "Constraint condition on transformed relation for generalized acoustics," e-print: arXiv:0912.5462.
[21] Z. Chang, J. Hu, and G. K. Hu, "Transformation method and wave control," Acta Mech. Sin. **26**, 889 (2010).