Invisible cloak design with controlled constitutive parameters and arbitrary shaped boundaries through Helmholtz's equation: comment

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Abstract: In a recent paper, Chen et al. [Opt. Express 17, 3581 (2009)] develop an approach to design invisible cloaks with controllable constitutive parameters by adjusting the constant k in the Helmholtz's equation. In this comment, we discuss the limitation of the free parameter *k* in designing cloaks. It is found that the real constant *k* can be chosen only as limited values in order to avoid the singular material parameters.

OCIS codes: (230.3205) Invisibility cloaks; (160.1190) Anisotropic optical materials; (230.0230) Optical devices.

References and links

1. J. Hu, X. Zhou, and G. Hu, "Design method for electromagnetic cloak with arbitrary shapes based on Laplace's equation," Opt. Express **17**(3), 1308–1320 (2009).
2. J. B. Pendry, D. Schurig, and D. R. Smith, "Controlling electromagnetic fields," Science **312**(5781), 1780–1782 (2006).
3. X. Chen, Y. Fu, and N. Yuan, "Invisible cloak design with controlled constitutive parameters and arbitrary shaped boundaries through Helmholtz's equation," Opt. Express **17**(5), 3581–3586 (2009).

Recently, Hu et al [1] present a method to design arbitrary cloaks based on the deformation perspective of transformation optics [2]. In this method, there are no free controllable parameters to tune material parameters of cloaks. In a recent paper, Chen et al [3] develop this idea to design invisible cloaks of arbitrary shapes with controllable constitutive parameters by introducing a constant k from the Helmholtz equation $\nabla^2\varphi + k^2\varphi = 0$. In this comment, we will point out that there is the underlying limitation on the choice of the constant k and the simulation given in [3] is incorrect.

The Helmholtz equation with Dirichlet boundary conditions has time harmonic solutions when k is a real value. For a spherical cloak bordered by $r \in (r_1, r_2)$, there must be a series of local maximum and minimum values within the cloak region, if k, the propagating constant, is taken as a large real value. When these solutions are used, the principle stretch in the radial direction $\lambda_r = dr'/dr$ will be zero at the local extremum points. When the inverse form of Helmholtz equation is solved, the stretch λ_r will instead tend to be infinity at those points, while λ_θ and λ_ϕ are both finite. Since material parameters of the spherical cloak are given by $\varepsilon_r' = \mu_r' = \lambda_r / (\lambda_\theta \lambda_\phi)$ and $\varepsilon_\theta' = \mu_\theta' = \varepsilon_\phi' = \mu_\phi' = \lambda_\theta / (\lambda_r \lambda_\phi)$ [1], singular material parameters within the spherical cloak will occur, as is unexpected and will greatly increase the difficulty of practical realization. Under this consideration, the real value k must be chosen in a small-value region. Figure 1(a) gives the transformation relations $r'(r)$ for the spherical cloaks with $r_1 = 0.2$ and $r_2 = 0.25$ by calculating the inverse form of Helmholtz equation with different k. Figure 1(b) shows the corresponding radial permittivities ε_r, which will be singular when the cloak region involves finite extremum points.

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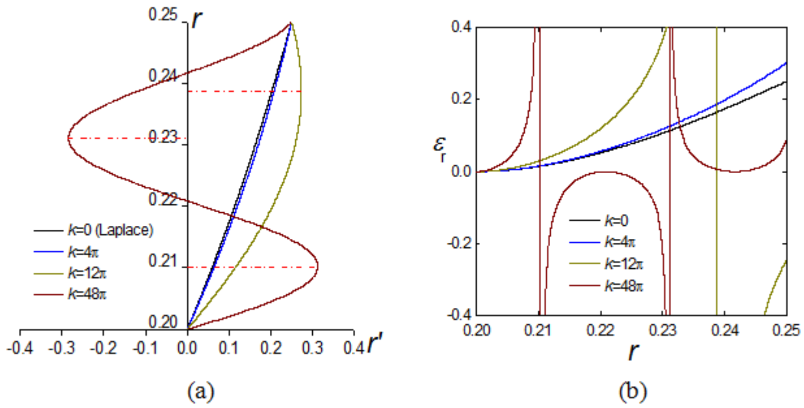

Fig. 1. (a) The transformation relation $r'(r)$ for spherical cloaks with r_1 = 0.2 and r_2 = 0.25 designed by different k in the Helmholtz equation. (b) The radial permittivity ε_r of spherical cloaks corresponding to the transformation given in (a).

In performing FEM simulations (Fig. 3 in Ref [3].), Chen et al [3] might make a mistake that they wrongly set k as imaginary values. For supporting this point, we recalculate those examples and show the comparison results for real (left) and imaginary (right) k in Fig. 2. It is clearly seen that Chen's results correspond to the imaginary k, while singular material parameters will exist when k takes real values, like $k = 30\pi$ and $k = 50\pi$. Fortunately, the imaginary k means the evanescent-wave type of the transformation. There are no extremum points for the transformed displacement in this case, and hence no singular problem.

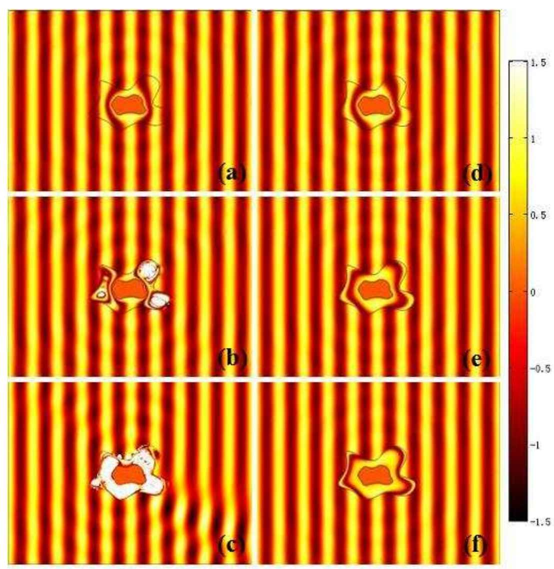

Fig. 2. Distribution of electric field near the cloak with different k. The white region denotes the area where the field is beyond the range of the color bar. (a) $k = 10\pi$. (b) $k = 30\pi$. (c) $k = 50\pi$. (d) $k = 10\pi i$. (e) $k = 30\pi i$. (f) $k = 50\pi i$.

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